

PULSE DETECTION IN SYNCOPATED RHYTHMS USING NEURAL OSCILLATORS

Marc J. Velasco

Edward W. Large

Center for Complex Systems and Brain Sciences

Florida Atlantic University

velasco@ccs.fau.edu, large@ccs.fau.edu

ABSTRACT

Pulse and meter are remarkable in part because these perceived periodicities can arise from rhythmic stimuli that are not periodic. This phenomenon is most striking in syncopated rhythms, found in many genres of music, including music of non-Western cultures. In general, syncopated rhythms may have energy at frequencies that do not correspond to perceived pulse or meter, and perceived metrical frequencies that are weak or absent in the objective rhythmic stimulus. In this paper, we consider syncopated rhythms that contain little or no energy at the pulse frequency. We used 16 rhythms (3 simple, 13 syncopated) to test a model of pulse/meter perception based on nonlinear resonance, comparing the nonlinear resonance model with a linear analysis. Both models displayed the ability to differentiate between duple and triple meters, however, only the nonlinear model exhibited resonance at the pulse frequency for the most challenging syncopated rhythms. This result suggests that nonlinear resonance may provide a viable approach to pulse detection in syncopated rhythms.

1. INTRODUCTION

Pulse is a periodicity perceived in a musical rhythm, operationally defined as the frequency at which one would most likely tap along to a rhythm [11]. People also perceive meter, a structural pattern of accents among beats of the pulse [10]. Pulse and meter can be diagrammed using the notation of Lerdahl and Jackendoff [10], in which the metrical grid is composed of beats at multiple related frequencies, with strong beats occurring when beats at multiple frequencies overlap in time. Thus meter organizes beats of the pulse into strong beats and weak beats.

In simple rhythms (Figure 1a), note-events occur on strong beats. Rhythms such as the 3-2 Rumba Clave (Figure 1b), although they share the same nominal metrical structure, are more complex. In such rhythms, note-events occur on metrically weak beats, and strong metrical beats

often correspond to silences. These two attributes define syncopation [3, 12]. Thus, in syncopated rhythms note events are spaced irregularly in time, yet the perceived pulse is regularly timed, and the meter, regularly structured [3, 14]. A goal of theories of pulse perception is to explain how pulse and meter are perceived for musical rhythms in general, and for syncopated rhythms in particular.

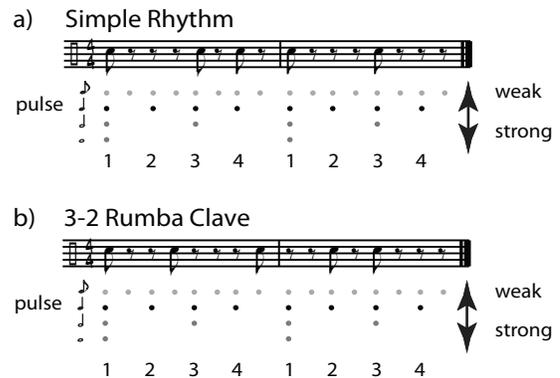


Figure 1. Example rhythms and metrical grid.

Our approach is based on the idea that the pulse perceived in a musical rhythm is a neural resonance that arises in sensory [6, 8, 17] and motor cortices [2, 4]. The experience of meter is posited to arise from interaction of neural resonances at different frequencies. In this paper we put forth a neurodynamic model of pulse and meter and ask whether it can explain the perception of pulse and meter in highly syncopated rhythms.

1.1 Neural Oscillation

Neural oscillation can arise from the interaction between excitatory and inhibitory neural populations. The canonical model used here was derived, using normal form theory, from the Wilson-Cowan model of the interaction between excitatory and inhibitory neural populations [7, 18]. This model is generic, however, so the responses of the model to musical rhythm are likely to be observed in many other nonlinear oscillator models of rhythm perception.

1.2 Model

Our conceptual model is a network of neural oscillators, spanning a range of natural frequencies, stimulated with an auditory rhythm. The basic concept is similar to signal processing by a bank of linear filters [15], but with the important difference that the processing units are nonlinear, rather than linear resonators.

We can describe the behavior of a linear filter using a differential equation (Eq 1), where the overdot denotes differentiation with respect to time. z is a complex-valued state variable; ω is radian frequency. $\alpha < 0$ is a linear damping parameter. $x(t)$ denotes linear forcing by a time-varying external signal.

$$\dot{z} = z(\alpha + i\omega) + x(t) \quad (1)$$

Because z is a complex variable, it has both amplitude and phase. Resonance in a linear system means that the system oscillates at the frequency of stimulation, with amplitude and phase determined by system parameters. As stimulus frequency, ω_0 , approaches the oscillator frequency, ω , oscillator amplitude, $r = |z|$, increases, providing band-pass filtering behavior. In the linear case, oscillator amplitude depends linearly on stimulus amplitude.

A common model of nonlinear oscillation is based on the normal form for the Hopf bifurcation (Eq 2).

$$\dot{z} = z(\alpha + i\omega + \beta|z|^2) + x(t) + h.o.t. \quad (2)$$

Note the surface similarities between this form and the linear resonator of Equation 1. Equation 2 can be seen as a generalization of Equation 1, and the two behave the same when $\beta = 0$. Again ω is radian frequency, and α is still a linear damping parameter. $\beta < 0$ is a nonlinear damping parameter, which maintains stability when $\alpha > 0$. $x(t)$ denotes linear forcing by an external signal. The term *h.o.t.* denotes higher-order terms of the nonlinear expansion that are truncated (i.e., ignored) in normal form models. When $\alpha = 0$ and $\beta < 0$, the system is said to be in the *critical* parameter regime, poised between damped and spontaneous oscillation. The amplitude of the response depends nonlinearly on the input amplitude. Like linear resonators, nonlinear oscillators have a filtering behavior, responding maximally to stimuli near their own frequency. Differences in behavior include extreme sensitivity to weak signals and high frequency selectivity. Critical oscillators have been

used to model critical oscillations of outer hair cells in the cochlea [5]. When $\alpha > 0$ (and $\beta < 0$), the system exhibits a limit cycle in absence of input; thus, it can oscillate spontaneously.

Our canonical model [7] (Eq 3) is an expansion of the Hopf normal form (Eq 2), which includes higher order terms.

$$\dot{z} = z(\alpha + i\omega + (\beta_1 + i\delta_1)|z|^2 + \frac{(\beta_2 + i\delta_2)\epsilon|z|^4}{1 - \epsilon|z|^2}) + c P(\epsilon, x(t))A(\epsilon, \bar{z}) \quad (3)$$

There are again surface similarities with the previous models. The parameters, ω , α and β_1 correspond to the parameters of the truncated model. β_2 is an additional amplitude compression parameter, and c represents strength of coupling to the external stimulus. δ_1 and δ_2 are frequency detuning parameters. The parameter ϵ controls the amount of nonlinearity in the system. Most importantly, coupling to a stimulus is nonlinear and has a passive part, $P(\epsilon, x(t))$ and an active part, $A(\epsilon, z)$, as defined in [7], which produce different higher order resonances, as described in the next section.

1.3 Properties of Nonlinear Resonance

Equation 3 displays all the behavioral regimes described above – linear, critical and limit cycle – depending on the parameter values chosen. Additionally, Equation 3 can also exhibit a double-limit cycle bifurcation, when $\alpha < 0$, $\beta_1 > 0$, $\beta_2 < 0$ (and $\epsilon > 0$). Stable states emerge at rest and at a stable limit cycle; an unstable limit cycle separates the two, functioning as a kind of threshold. If the stimulus is strong enough, the threshold will be crossed, the system reaches the stable limit cycle, and oscillation can be maintained even after the stimulus has ceased. Thus an oscillator operating in a double-limit cycle regime can maintain a memory of an oscillating stimulus.

Higher-order resonance means that a nonlinear oscillator with frequency f responds to harmonics ($2f, 3f, \dots$), subharmonics ($f/2, f/3, \dots$) and integer ratios ($2f/3, 3f/4, \dots$) of f . If a stimulus contains multiple frequencies, a nonlinear oscillator will respond at combination frequencies ($f_2 - f_1, 2f_1 - f_2, \dots$) as well. Higher order resonances follow orderly relationships and can be predicted given stimulus amplitudes, frequencies and phases. This has important implications for understanding the behavior of such systems. The nonlinear oscillator network does not merely transduce signals; it adds frequency information, which can be used to model pattern recognition and pattern completion, among other things. Neural pattern completion based on nonlinear resonance may explain the perception of pulse and meter in syncopated rhythmic patterns [9, 13].

Our hypothesis is that in rhythms with no energy at the pulse frequency, pulse arises due to nonlinear resonance in the brain. Significant contributions may also come from intrinsic dynamics and learned connectivity. As a first test of this hypothesis, we ask whether such resonances arise in a canonical nonlinear model.

2. EXPERIMENT 1

The first experiment compared the objective frequency content of 16 rhythms with the frequency responses of a nonlinear oscillator network. Using Fourier analysis we measured the frequency content of the rhythmic patterns, showing that in syncopated rhythms the pulse frequency is weak or absent. Next, we assessed whether nonlinear resonance could explain the perception of pulse and meter at the frequencies that are predicted by music theoretic analysis of these rhythms.

2.1 Model

Our model consisted of a single network of 289 oscillators described by Equation 3, with natural frequencies logarithmically spaced from 0.25 Hz to 16 Hz. The model operated in a critical parameter regime ($\alpha = 0$, $\beta_1 = -1$, $\beta_2 = -0.25$, and $\varepsilon = 1$), poised between damped and spontaneous oscillation.

2.2 Stimuli

We used 16 rhythms: one isochronous pulse train, two canonical metrical rhythms (3/4 and 4/4), three clave rhythms, and ten “missing pulse” rhythms that were created in our lab in the context of a previous experiment [1]. The clave rhythms were a 3-2 Son Clave, a Rumba Clave, and a clave-like rhythm we dubbed ‘Hard Clave’. The ten missing pulse rhythms were structured so as to balance strong and weak beats, with four events on strong beats and four events on weak beats. In a previous experiment we observed that most people reliably tap at the nominal pulse frequency for these rhythms. We rendered each rhythmic event as a continuous time onset ‘bump’ with amplitude corresponding to the intensity of the event. All events were of equal intensity, except for the metrical rhythms, where intensity differences marked canonical metrical accents. All rhythms were rendered at a tempo of 120 bpm, making the pulse frequency 2 Hz. Examples of the rhythmic stimuli are shown in Figure 2.

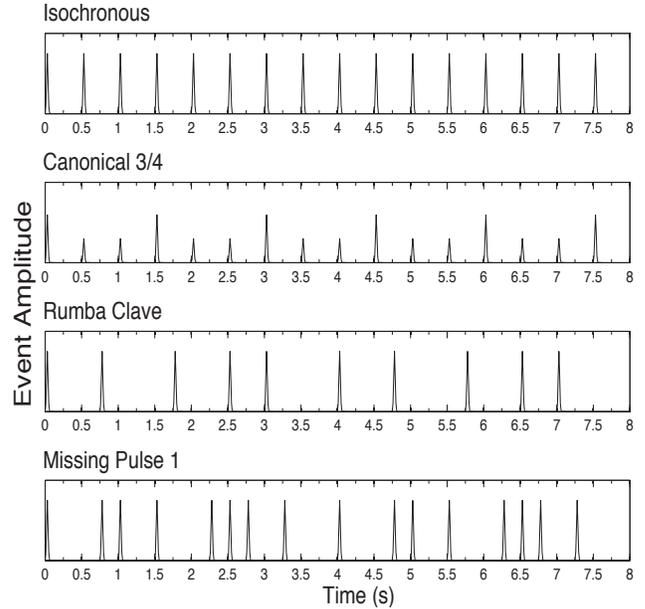


Figure 2. Examples of stimuli types: Isochronous, Canonical 3/4, a Rumba Clave, and one of the ten missing pulse patterns.

2.3 Method

Computations were performed using Matlab 7.4, on a Macintosh Mac Pro, running Mac OS X 10.5.8. In the simulations, the continuous-time pulse trains were used to drive the network model and the resulting oscillatory output behavior was examined. Network behavior was evaluated by assessing steady state amplitude of the resonating oscillators. The natural frequencies of the resonating oscillators indicate which frequencies resonate to the input stimulus.

2.4 Results

Figure 3 compares a Fourier analysis (FFT) of four rhythmic input signals with the amplitude profile of the network of nonlinear oscillators. Oscillator natural frequency (Hz) runs along the x-axis, and amplitude is shown on the y-axis. Musical notation above each panel indicates the pulse and metrical frequencies for each rhythm. For the isochronous rhythm, energy is present at the pulse frequency (2 Hz), and its harmonics. For the canonical rhythms, signal energy was observed at the pulse frequency, while the accents present in the signal contributed frequencies at metrical levels (subharmonics of the pulse). The clave rhythms all had some energy at 2 Hz; however, this was strongly attenuated compared to the energy at other nearby frequencies. Fourier analysis of the other ten syncopated rhythms revealed no energy at the 2 Hz pulse frequency, while considerable energy was

observed at non-metrical frequencies. Note that energy was present at the eighth note level of 4 Hz for all rhythms.

As illustrated in Figure 3, resonant responses were observed in the oscillator network at frequencies that were not objectively present in the stimulus rhythms. Most importantly, resonances were observed at the pulse frequency for every rhythm. Resonances were also observed at subharmonics of the isochronous rhythm, and for canonical rhythms subharmonic resonances enhanced the response at the metrical frequencies. For the clave rhythms, the response at the pulse frequency (2 Hz) was also enhanced relative to the Fourier amplitude. For the missing pulse rhythms, although there was no energy at the 2 Hz pulse frequency, the nonlinear network responded at the 2 Hz pulse frequency as well as at some additional metrical frequencies.

In summary, both simple and complex rhythms contain multiple frequencies, only some of which appear to be related to the meter. Simple rhythms contain frequencies corresponding to the pulse; however, complex syncopated rhythms contain little or no energy at the pulse frequency. This feature of complex rhythms may be problematic for linear filter based methods of pulse detection. Nonlinear oscillators can resonate at frequencies corresponding to pulse and meter even when these are not objectively present in the input. However, the simple oscillator array investigated in Experiment 1 is, by itself, likely not sufficient to induce the pulse and meter of complex rhythms. While oscillators resonate at the pulse frequency, a number of stronger resonances are observed at frequencies that do not correspond to pulse or meter. In the next experiment, we ask whether multiple networks together might provide greater frequency selectivity.

3. EXPERIMENT 2

3.1 Stimuli & Method

The stimuli methods used in Experiment 2 were the same as in Experiment 1.

3.2 Model

The model was based on the same oscillator equations as used in Experiment 1. The key difference was that in Experiment 2, the model consisted of two networks interacting with each other. Network 1 had the same parameters as used in Experiment 1. The oscillators in Network 2 were tuned to exhibit double limit cycle bifurcation behavior ($\alpha = 0.3$, $\beta_1 = 1$, $\beta_2 = -1$, and $\varepsilon = 1$), and thus exhibited both threshold and memory properties.

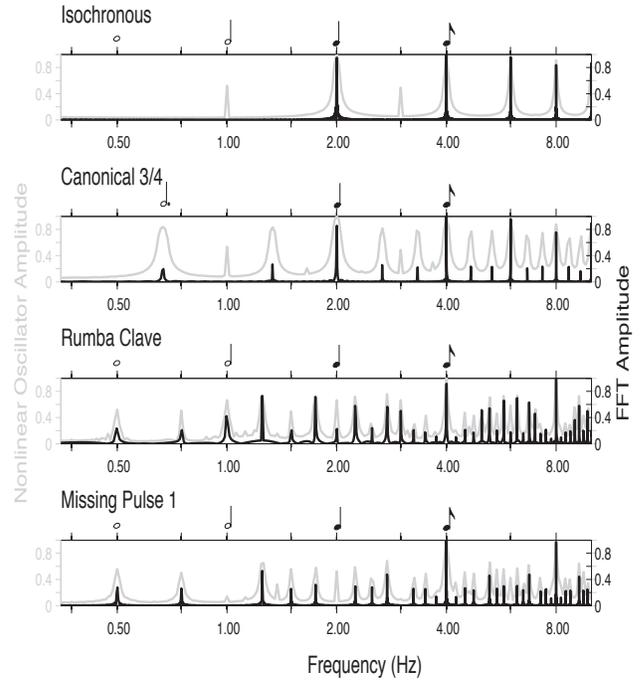


Figure 3. Experiment 1 results. A subset of the rhythms presented with both an FFT of the stimulus (black) and the amplitudes of responding nonlinear oscillators (gray).

The two networks were connected as shown in Figure 4. Tonotopic connections between the networks allow Network 1 to drive Network 2. Next, in each network, internal connectivity coupled patches of oscillators to other patches exhibiting small integer ratio frequency relationships, 1:3, 1:2, 1:1, 2:1, 3:1. These connections are assumed to be learned by exposure to Western rhythms, in which duplet and triplet meters are common. Connectivity from Network 2 to Network 1 was inhibitory.

3.3 Results

Across the rhythms presented, Network 1 behaved similarly to the previous experiment, responding to frequencies present in the stimulus rhythms, and also adding nonlinear resonances. Example of Network 2 responses are shown in Figure 5. Due to its thresholding properties, Network 2 responded to a subset of frequencies present in the Network 1. Importantly, Network 2 almost always responded at the pulse frequency. Moreover, the amplitude at 2 Hz was unexpectedly strong given the relatively weak responses observed in Experiment 1.

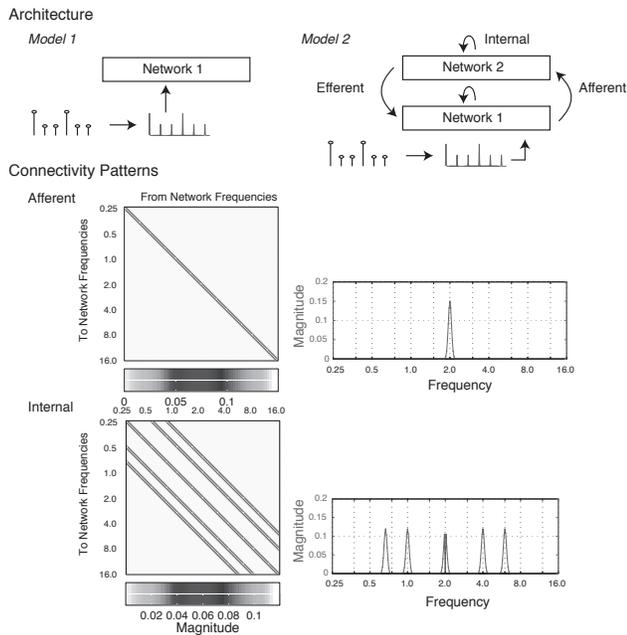


Figure 4. Network architecture for models used in both experiments.

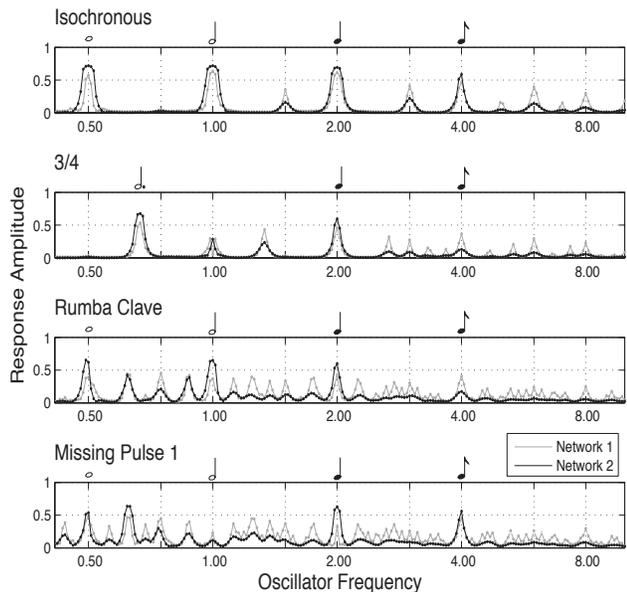


Figure 5. Results for Experiment 2. Amplitude response profiles for Network 1 (gray) and Network 2 (black).

Frequencies were considered ‘active’ in Network 2 if they exceeded the threshold implicit in the double limit cycle oscillatory dynamics. Active frequencies were compared to metrical frequencies for each rhythm. For syncopated rhythms expected frequencies were the quarter note level (i.e., the pulse, 2 Hz), the eighth note level (4 Hz), as well as the half note (1 Hz) and whole note levels (0.5 Hz) for

Stimuli Rhythm	Network 2 Active Frequencies (Hz)					Other
	1/2	2/3	1	2	4	
Isochronous	x		x	x	x	
4/4	x		x	x		
3/4		x		x		
Son Clave	x		x	x		0.62
Rumba Clave	x		x	x		0.62, 0.88
Hard Clave			x	x		0.62, 1.24
Missing Pulse 1	x			x	x	0.62
Missing Pulse 2				x	x	0.63, 0.75
Missing Pulse 3				x	x	0.62
Missing Pulse 4				x	x	0.62, 0.88, 1.12
Missing Pulse 5	x		x	x	x	0.62, 0.75, 1.24
Missing Pulse 6				x	x	0.62, 0.75
Missing Pulse 7	x			x		0.62, 0.75
Missing Pulse 8	x			x	x	0.75
Missing Pulse 9				x	x	0.75
Missing Pulse 10			x		x	0.63, 0.75, 1.26

Table 1. Summary of results for Experiment 2. Shaded cells identify frequencies which would be expected to have a resonance for the rhythm based on meter. Populated cells (x) show which resonant frequencies were active in Network 2.

most of the rhythms (the one exception was the canonical 3/4 rhythm, whose slower metrical frequency was 0.67 Hz). The results of the two-network model can be seen in Table 1. Highlighted cells show the frequencies at which response peaks would be expected based on the meter. Populated cells show whether or not response peaks were observed at given frequencies. For all but one rhythm, a response was seen at the pulse frequency of 2 Hz. For the canonical rhythms, response peaks were always found at the expected frequencies and at no others. This set of hierarchically related frequencies may correspond to a perception of meter. For the missing pulse rhythms, response peaks were found most consistently at the pulse frequency and its first harmonic at 4 Hz. At lower frequencies, the results differed from standard metrical predictions. This may explain why people sometimes have difficulty entraining periodic taps with highly syncopated stimuli. In previous experiments, level of syncopation was found to be a good predictor of pulse-finding difficulty; syncopation causes off-beat taps and some switches between on-beat and off-beat tapping [14, 16].

4. DISCUSSION

Syncopated rhythms present challenges for pulse detection algorithms. Looked at in the frequency domain, some syncopated rhythms do not contain any energy at the frequency of the pulse. Yet pulse is readily perceived in

syncopated rhythms [1, 14]. From the point of view of music perception, this observation implies that the brain adds frequency components that are not objectively present in rhythms themselves. A lack of energy at the pulse frequency may explain why pulse detection methods based on linear resonance experience problems with syncopated rhythms. For syncopated rhythms, our nonlinear model, based on fundamental principles of neurodynamics, resonates at the pulse frequency. This qualitatively matches human performance [1], and the detailed responses of this model provide novel predictions which could be tested in future experiments. Our observations support the hypothesis that pulse corresponds to a neural resonance. In simple networks, nonlinear resonance by itself is capable of restoring a missing pulse frequency. When multiple networks of nonlinear oscillators are coupled together (including internal rhythmic connectivity within networks), they can resonate at a pulse frequency and related metrical frequencies, a form of temporal pattern matching or pattern completion.

In future work, we plan to construct and test other models based on nonlinear resonance. For example, the results presented here do not enable us to say whether internal network connectivity or the thresholding properties of Network 2 were primarily responsible for the observed responses. Perhaps both are necessary. Future work in this area will focus on how the connectivity patterns between networks are learned and address developmental aspects of pulse and meter as well as differences across cultures.

5. REFERENCES

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