The Resonant Dynamics of Beat Tracking and Meter Perception

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Abstract. One of the basic processes that affords the perception of metrical structure is the ability to perceive and follow the *beat* of a piece of music. The apparent simplicity of this behavior, however, masks a notoriously complex aspect of the human response to musical rhythm. Computer simulation of human "beat-tracking" behavior has proved exceedingly difficult. Many of the problems for computer simulation stem from the fact that musicians never perform rhythms in a perfectly regular, or mechanical, fashion. Performers produce sound patterns that reveal both intentional and unintentional timing variability. While such timing variability rarely poses difficulty for human performers and listeners, it seems that once a computer program loses the beat, recovery is nearly impossible. This paper presents a new model of human beat-tracking behavior as a dynamic process in which the temporal organization of external musical events synchronizes, or *entrains*, a set of non-linearly coupled oscillators. I analyze the behavior of this model using geometric techniques, and discuss implications for the perception of metrical structure.

Introduction. The perception of rhythm in music involves, among other things, the perception of *metrical structure*. Metrical structure describes an important part of musical phenomenology, the sense of alternating strong and weak beats that accompanies the experience of listening to music (Lerdahl & Jackendoff, 1983). Lerdahl and Jackendoff (1983) have proposed a generative theory that describes the metrical structure of a piece as a grid, as shown in Figure 1. Each row of dots represents a level of beats, and the relative spacing between dots of adjacent levels describes the relationship between the beat periods of adjacent levels. Lerdahl and Jackendoff's (1983) metrical grids explicitly capture the perception of regularly recurring strong and weak beats. Points where the beats of many levels coincide are strong beats; points where few beats coincide are weak beats. This theory posits rules for describing legal metrical structures (e.g. Lerdahl & Jackendoff, 1983), and the perception of rhythm is thought to involve parsing a rhythmic pattern to retrieve its metrical structure (Jackendoff, 1992).

Figure 1: A metrical structure grid describing an alternation of strong and weak beats.

In musical performance, however, tempo changes over time (see, for example Clarke, 1985; Shaffer, Clarke, & Todd, 1985; Sloboda, 1983), complicating the task of meter perception. Two basic strategies have emerged to deal with the perception of beat and meter in the face of timing variability *beat-tracking* (see, for example Allen & Dannenberg, 1989; Dannenberg, 1984; Dannenberg & Mont- Reynaud, 1987; Longuet-Higgins 1987) and *quantization* (Desain & Honing; 1991). The model described here is a beat-tracking approach (Large & Kolen, in press). Unlike previous beat-tracking approaches, however, this approach to modeling the perception of metrical structure is based on the mathematics of coupled oscillation. It is consistent with psychological models of motor coordination (*cf.* Kelso & deGuzman, 1992; Schmidt, et. al., 1991; Shaffer, 1981; Treffner & Turvey, 1993) and rhythm perception (*cf.* Essens & Povel, 1985; Jones, 1976; Jones & Boltz, 1989; Povel & Essens, 1985). A rhythmic pattern serves as a driving signal. Through non-linear coupling, the rhythm perturbs both the phase and the intrinsic period of individual oscillators, modeling musical beat. A network of oscillators entrains to the periodic components of a rhythmic signal at different time-scales, and to the outputs of one another. Metrical structure is modeled as the collective consequence of mutual entrainment among many constituent processes (Large & Kolen, in press).

A Resonance Model. The rhythm is represented as a series of discrete pulses corresponding to the onset of individual musical events (e.g. notes). Each oscillator adjusts both its phase and period so that during stimulation the unit's output pulses track the phase and period of a driving rhythm. The output of the oscillator is given by:

$$o(t) = 1 + \tanh\left(\gamma\left(\cos\frac{2\pi}{p}(t - t_0) - 1\right)\right),$$

where t is time, p is the period of the oscillation, $(t-t_0)(mod)p$ is the phase, and γ is the output gain. Each output pulse instantiates a temporal receptive field for the oscillatory unit – a window of time during which the unit "expects" to see a stimulus pulse. The width of the receptive field can be adjusted by changing the unit's output gain, γ. The unit responds to stimulus pulses that occur within this field by adjusting its phase and period, according to delta rules that minimize the difference between when each unit expects an event, and when events actually occur.

Analysis. In this model, an output pulse fills the role of a beat. Output pulses differ from beats, however, in that each output pulse has a width. An event that occurs a little earlier or later than expected may still be interpreted an being "on the beat" and the oscillator will adjust its phase and period accordingly. Precisely how much deviation from timing regularity an oscillator will tolerate depends upon the parameters of the model. To understand the oscillator's behavior, it is useful to examine regime diagrams for the oscillator model. A regime diagram summarizes the response of a single oscillator to periodic stimulation, a simplified form of input that can be expected either from an external rhythm or from another oscillator in the network (Large & Kolen, in press). In the following regime diagrams, q is the period of the input (driver) and p is the period of the responding (driven) oscillator. The initial period ratio q/p, is plotted along the x-axis. The y-axis gives coupling strength, corresponding to the speed with which the oscillator adapts to tempo changes.

The regime diagrams show parameter regions that result in phase-locked states, called Arnol'd tongues (Glass & Mackey, 1988; Schroeder, 1991). Figure 2A shows the effects of phase-locking alone, Figure 2B shows the effect of tracking both phase and period. These diagrams show, for example, that even when q/p is different from 0.5, the coupled system may lock in a 1:2 relationship (the center "tongue" in the diagrams of Figure 2), because each time the driven oscillator fires, its phase and/or period is perturbed slightly by coupling to the driver. Each diagram identifies parameter regions that result in stable ratios, q/p. Non-integer ratios correspond to regions of polyrhythmic response. Darker regions reflect faster convergence on stable states.

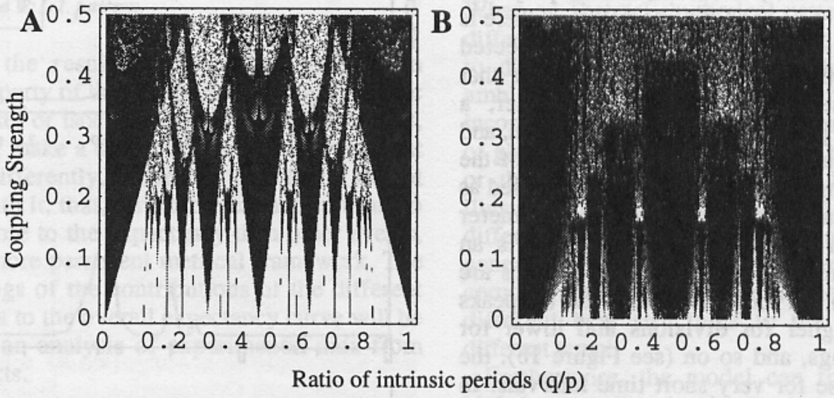

Figure 2: Regime diagrams describe stable phase-locking regions as a function of initial driver/driven period ratios and coupling strength. A) Phase-locking only, B) Frequency- and phase-locking. Each diagram shows the following regions (left to right): 0:1, 1:8, 1:7, 1:6, 1:5, 1:4, 2:7, 1:3, 3:8, 2:5, 7:3, 1:2, 4:7, 3:5, 5:8, 2:3, 5:7, 3:4, 4:5, 5:6, 6:7, 7:8, 1:1.

The width of each Arnol'd tongue reflects the sensitivity of the coupled system to deviations in the q/p ratio. Thus, these Arnol'd tongues predict sensitivity of the model to timing deviation in musical performance. The response depends on the ratio of the intrinsic periods of the input and the responding oscillator, as well as on the values of the model parameters.

Conclusions. The model of beat-tracking and meter perception described in this paper uses oscillators that synchronize periodic output pulses with rhythmic patterns. The response of a single oscillator to a rhythmic pattern models beat-tracking behavior. The response of a network of oscillators to a rhythmic pattern models the perception of meter. Analysis of the behavior of a single oscillatory unit in response to a simple periodic signal reveals complex non-linear dynamics, and important differences compared with previous models. Previous models of beat-tracking may have underestimated the dynamic complexity of the beat-tracking task, leading to unstable behavior. Regime diagrams describe the complex behavior of a single oscillator, permitting network design and choice of model parameters that enable stable beat-tracking and meter perception. Models based on this formalism handle performance timing and polyrhythmic structure.