Tonality and Nonlinear Resonance

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ABSTRACT: We outline a theory of tonality that predicts tonal stability, attraction, and categorization based on the principles of nonlinear resonance. Perception of tonality is the natural consequence of neural resonance, arising from central auditory nonlinearities.

KEYWORDS: auditory processing; tonality; hearing; cochlea

INTRODUCTION

Tonality is the organized relationship of tones in music.[1] A tonal system includes a central tone, or tonic, and a collection of related tones, each with a prespecified fundamental frequency. Within a tonal context, certain tones are perceived as more stable than others, such that less stable tones provide points of stress, and more stable tones provide points of repose. Less stable tones are heard relative to more stable ones, such that more stable tones are said to attract less stable tones.[2] What nervous system processes give rise to such perceptions in music?

Recent evidence suggests that the cochlea performs active frequency transformation of sounds by nonlinear resonance, using a network of locally coupled outer–hair cell oscillators.[3,4] In species that lack cochleae, auditory neurons respond selectively to temporal and spectral features of communication sounds,[5] and nonlinear resonance based on excitation and inhibition has been implicated in such responses.[6] In the mammalian auditory system, neural activity in several areas, including the cochlear nucleus, the inferior colliculus, and A1, is phase locked to the stimulus waveform. Inhibition plays a significant role in these responses.[7] Phase locking deteriorates as the auditory pathway is ascended.[8]

HYPOTHESIS

We hypothesize that nonlinear frequency transformation takes place in the mammalian central auditory nervous system (CANS). Nonlinear frequency transformation occurs when a network of coupled nonlinear resonators, each tuned to a distinct eigenfrequency, is driven by an external stimulus. A nonlinear neural resonator can

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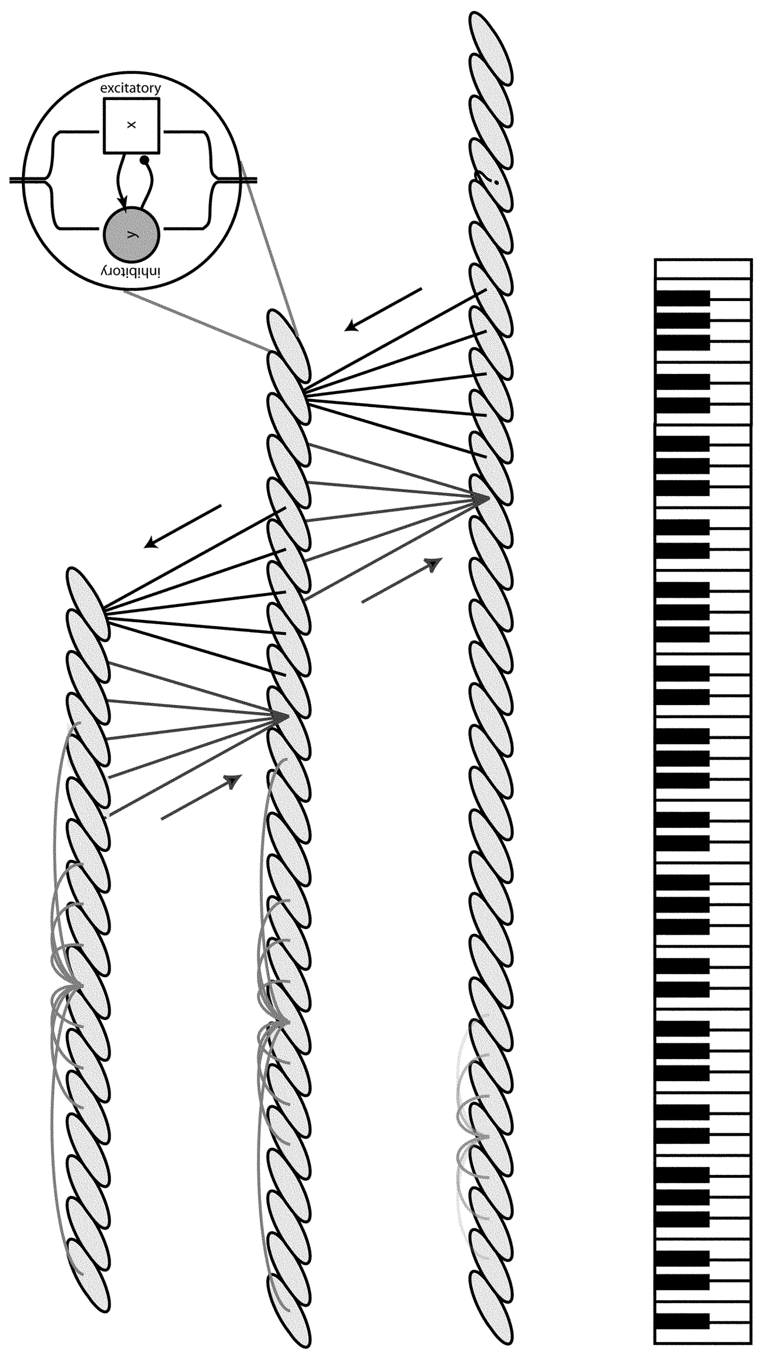

FIGURE 1. Cascaded nonlinear frequency transformation. Initial analysis is performed by the cochlea, and further transformations are performed in the CANS by neural oscillators. Phase locking deteriorates as the pathway is ascended. **Inset**: A neural oscillator, consisting of an excitatory-inhibitory neuron pair.

be modeled as a pair of interacting neurons (or populations), one excitatory and one inhibitory.[9,10] Mathematical analysis of resonator networks reveals generic properties of nonlinear frequency transformation, including extreme sensitivity to weak stimuli, sharp frequency tuning, amplitude compression, frequency detuning, natural phase differences, and nonlinear distortions.[11] These properties are consistent with psychoacoustic phenomena, such as hearing thresholds, frequency discrimination, loudness scaling, Stevens' rule, harmonic distortion, and combination tones. Nonlinear resonance is also a plausible neural mechanism for pitch perception in humans.[12]

A simple nonlinear CANS model is shown in FIGURE 1. After initial nonlinear frequency analysis by the cochlea, networks of neural resonators further transform the stimulus. Such a cascaded analysis is not redundant, because nonlinear transformations cannot be collapsed into a single stage, as is the case with linear transformations. A recent analysis suggests that as the series of nonlinear transformations is ascended, a complex web of resonances arises, including objective stimulus frequencies and a large number of nonlinear "distortions."[13] A dynamic field emerges, with active resonances in various regions embodying a musical scale.

PREDICTIONS

The Arnol'd tongues bifurcation diagram (FIG. 2) allows the study of these resonances, by postulating a specific connectivity.[13] The horizontal axis gives the ratio of each oscillator's frequency to the tonic, while the vertical gives the strength of afferent input at the tonic frequency. The resonance regions, or tongues, show the neural areas that will resonate for various input strengths. As stimulus strength is increased, larger patches of neurons resonate, indicated by the increasing width of the tongues. Moreover, each area resonates at a specific frequency ratio with the tonic.

The widest tongues reflect the most stable resonances. A stable resonance is resistant to change in frequency due to interactions with other parts of the dynamic field. The relative stability of tones in the key of C major is predicted by the relative widths of the resonance regions in the bifurcation diagram. The theory also makes predictions about tonal attraction. In regions where the tongues overlap, more stable resonances tend to overpower less stable ones, such that the neural population in the overlap region will tend to oscillate at the frequency of the more stable resonance.

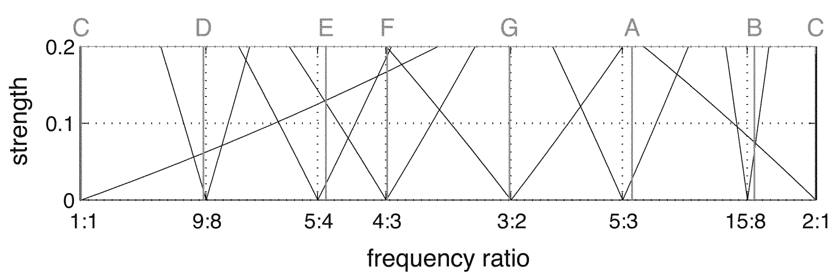

FIGURE 2. Arnol'd tongues diagram showing predicted resonances for the major scale. The width of the resonance regions predicts stability of the corresponding tones. Overlapping of regions predicts tonal attraction, with more stable regions dominating.

Historical notions of consonance and dissonance hold that musical consonance is determined by the ratios of small whole numbers.[14] However, the principle of small integer ratios does not explain the acceptability of equal temperament, because equal-tempered (ET) intervals are irrational. Also it does not explain the acceptability of large variations in intonation commonly observed in musical performance.[15] Perceptual categorization of musical intervals[15] explains both observations, but contradicts the principle of small integer ratios.

The gray vertical lines in FIGURE 2 denote ET frequencies. Resonance center frequencies do not precisely match ET tone frequencies. However, as stimulus strength increases, larger neural populations resonate, eventually encompassing the equal tempered ratios. Our theory predicts both perceptual categorization and the importance of small integer ratios. Resonance predicts how much variation in intonation is acceptable before a pitch is heard as mistuned.

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[Competing interests: The authors declare that they have no competing financial interests.]

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